1 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{R})\ (x \in \mathbb{C})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

2 DeMorgan's Law

Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

3 XOR

The truth table of XOR is as follows.

A	B	A XOR B
F	F	F
F	T	T
T	F	T
T	T	F

1. Express XOR using only $(\wedge, \vee, \neg)$ and parentheses.
2. Does $(A \text{ XOR } B)$ imply $(A \vee B)$? Explain briefly.
3. Does $(A \vee B)$ imply $(A \text{ XOR } B)$? Explain briefly.

4 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

(b) $\exists x \exists y P(x,y) \implies \exists y \exists x P(x,y)$.

(c) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

(d) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.