CS 70 Discrete Mathematics and Probability Theory
Summer 2019 James Hulett and Elizabeth Yang

DIS 4A

1 Countability and the Halting Problem

Prove the Halting Problem using the set of all programs and inputs

a) Show that the set of all programs are countable.

b) Show that the set of all inputs are countable.

c) Assume that you have a program that tells you whether or not it halts. Since the set of all programs and the set of all inputs are countable, we can enumerate them and construct the following table.

	x_1	x_2	x_3	x_4	...
p_1	H	L	H	L	...
p_2	L	L	L	H	...
p_3	H	L	H	L	...
p_4	L	H	L	L	...
⋮	⋮	⋮	⋮	⋮	⋱

Now write a program that is not within the set of programs in the table above.

d) Find a contradiction in part a and part b to show that the halting problem can't be solved.

2 Computability

Decide whether the following statements are true or false. Please justify your answers.

(a) The problem of determining whether a program halts in time 2^{n^2} on an input of size n is undecidable.

(b) There is no computer program `Line` which takes a program P, an input x, and a line number L, and determines whether the L^{th} line of code is executed when the program P is run on the input x.