

1 Hello World!

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

- (a) You want to determine whether a program P on input x prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

- (b) You want to determine whether a program P prints "Hello World!" before running the k th line in the program. Is there a computer program that can perform this task? Justify your answer.

- (c) You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task? Justify your answer.

2 Clothes and Stuff

- (a) Say we've decided to do the whole capsule wardrobe thing and we now have only 5 different items of clothing that we wear (jeans, tees, shoes, jackets, and floppy hats, etc.). We have 3 variations on each of the items, and we wear one of each item every day. How many different outfits can we make?

- (b) It turns out 3 floppy hats really isn't enough of a selection, so we've bought 11 more, and we now have 14 floppy hats. Now how many outfits can we make?

- (c) If we own k different items of clothing, with n_1 variations of the first item, n_2 variations of the second, n_3 of the third, and so on, how many outfits can we make?
- (d) We love our floppy hats so much that we've decided to also use them as wall art, so we're picking 4 of our 14 hats to hang in a row on the wall. How many such arrangements could we make? (Order matters.)
- (e) Ok, now we're packing for vacation to Iceland, and we only have space for 4 of our 14 floppy hats. How many sets of 4 could we bring? (Yeah, yeah, we knew you were going to use that notation. Now tell us the number as a function of d , your answer from the previous part.)
- (f) Ok, turns out the check-in person for our flight to Iceland is being *very* unreasonable about the luggage weight restrictions, and we're going to have to leave some hats behind. Despite our best intentions, and having packed only 4 hats, we actually bought 18 additional floppy hats at the airport (6 in burgundy, 6 in forest green, and 6 in classic black). We'll keep our 4 hats that we brought from home, but we'll have to return all but 6 of the airport hats. How many color configurations can there be for the 6 airport hats that we keep?

3 Bit String

How many bit strings of length 10 contain at least five consecutive 0's?