CS 70 Discrete Mathematics and Probability Theory
Summer 2019 James Hulett and Elizabeth Yang

DIS 4D

1 Venn Diagram

Out of 1000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.

(a) Suppose we choose a student uniformly at random. Let C be the event that the student belongs to a club and P the event that the student works part time. Draw a picture of the sample space Ω and the events C and P.

(b) What is the probability that the student belongs to a club?

(c) What is the probability that the student works part time?

(d) What is the probability that the student belongs to a club AND works part time?

(e) What is the probability that the student belongs to a club OR works part time?

2 Parking Lots

Some of the CS 70 staff members founded a start-up company, and you just got hired. The company has twelve employees (including yourself), each of whom drive a car to work, and twelve parking spaces arranged in a row. You may assume that each day all orderings of the twelve cars are equally likely.

(a) On any given day, what is the probability that you park next to Professor Rao, who is working there for the summer?

(b) What is the probability that there are exactly three cars between yours and Professor Rao's?

(c) Suppose that, on some given day, you park in a space that is not at one of the ends of the row. As you leave your office, you know that exactly five of your colleagues have left work before you. Assuming that you remember nothing about where these colleagues had parked, what is the probability that you will find both spaces on either side of your car unoccupied?

3 Calculate These... or Else

(a) A straight is defined as a 5 card hand such that the card values can be arranged in consecutive ascending order, i.e. $\{8,9,10,J,Q\}$ is a straight. Values do not loop around, so $\{Q,K,A,2,3\}$ is not a straight. However, an ace counts as both a low card and a high card, so both $\{A,2,3,4,5\}$ and $\{10,J,Q,K,A\}$ are considered straights. When drawing a 5 card hand, what is the probability of drawing a straight from a standard 52-card deck?

(b) What is the probability of drawing a straight or a flush? (A flush is five cards of the same suit.)

(c) When drawing a 5 card hand, what is the probability of drawing at least one card from each suit?

(d) Two distinct squares are chosen at random on 8×8 chessboard. What is the probability that they share a side?

(e) 8 rooks are placed randomly on an 8×8 chessboard. What is the probability none of them are attacking each other? (Two rooks attack each other if they are in the same row, or in the same column.)