1 Boy or Girl Paradox

You know Mr. Smith has two children, at least one of whom is a boy. Assume that gender is independent and uniformly distributed, so for any child, the probability that they are a boy is the same as the probability they are a girl, which is $1/2$.

(a) What is the probability that both children are boys?

(b) Now suppose you knock on Mr. Smith's front door and you are greeted by a boy who you correctly deduce to be Mr. Smith's older child. What is the probability that he has two boys? Compare your answer to the answer in part (a).

2 Box of Marbles

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

(a) If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?

(b) If we see that the marble is blue, what is the probability that it is chosen from box 1?

(c) Suppose we pick one marble from box 1 and without looking at its color we put it aside. Then we pick another marble from box 1. What is the probability that the second marble is blue?

3 Lie Detector

A lie detector is known to be 4/5 reliable when the person is guilty and 9/10 reliable when the person is innocent. If a suspect is chosen from a group of suspects of which only 1/100 have ever committed a crime, and the test indicates that the person is guilty, what is the probability that he is innocent?

4 Bayes Rule – Man Speaks Truth

(a) A man speaks the truth 3 out of 4 times. He flips a biased coin that comes up Heads 1/3 of the time and reports that it is Heads. What is the probability it is Heads?

(b) A man speaks the truth 3 out of 4 times. He rolls a fair 6-sided die. When you ask him if the die came up with a 6, he answers "yes". What is the probability it is really 6?

5 Weathermen

Tom is a weatherman in New York. On days when it snows, Tom correctly predicts the snow 70% of the time. When it doesn't snow, he correctly predicts no snow 95% of the time. In New York, it snows on 10% of all days.

(a) If Tom says that it is going to snow, what is the probability it will actually snow?

(b) What is Tom's overall accuracy?

(c) Tom's friend Jerry is a weatherman in Alaska. Jerry claims that she is a better weatherman than Tom even though her overall accuracy is lower. After looking at their records, you determine that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny day. Give an instance of the situation described above. *Hint: what is the weather like in Alaska?*