CS 70 Discrete Mathematics and Probability Theory
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DIS 5B

1 Mutually Independent Events

There are three mutually independent events: A, B, and C. The probability that event A occurs is 0.4, the probability that event B occurs is 0.6, and the probability that event C occurs is 0.3. Calculate the following.

(a) $\mathbb{P}([A|B]$.

(b) $\mathbb{P}[A \cap B]$.

(c) $\mathbb{P}[A \cup C]$.

(d) $\mathbb{P}[B \cap C]$.

(e) $\mathbb{P}[A \cap B \cap C]$.

(f) $\mathbb{P}[A \cup B \cup C]$.

2 Let's Talk Probability

(a) When is $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ true? What is the general rule that always holds?

(b) When is $\mathbb{P}(A \cap B) = \mathbb{P}(A) * \mathbb{P}(B)$ true? What is the general rule that always holds?

(c) If A and B are disjoint, does that imply they're independent?

3 Unlikely Events

(a) Toss a fair coin x times. What is the probability that you never get heads?

(b) Roll a fair die x times. What is the probability that you never roll a six?

(c) Suppose your weekly local lottery has a winning chance of $1/10^6$. You buy lottery from them for x weeks in a row. What is the probability that you never win?

(d) How large must x be so that you get a head with probability at least 0.9? Roll a 6 with probability at least 0.9? Win the lottery with probability at least 0.9?

4 Balls and Bins

You have n empty bins and you throw balls into them one by one randomly. A collision is when a ball is thrown into a bin which already has another ball.

(a) What is the probability that the first ball thrown will cause the first collision?

(b) What is the probability that the second ball thrown will cause the first collision?

(c) What is the probability that, given the first two balls are not in collision, the third ball thrown will cause the first collision?

(d) What is the probability that the third ball thrown will cause the first collision?

(e) What is the probability that, given the first $m-1$ balls are not in collision, the m^{th} ball thrown will cause the first collision?

(f) What is the probability that the m^{th} ball thrown will cause the first collision?

5 Pairwise Independence

The events A_1, A_2, A_3 are *pairwise independent* if, for all $i \neq j$, A_i is independent of A_j. However, pairwise independence is a weaker statement than *mutual independence*, which requires the additional condition that $\mathbb{P}(A_1, A_2, A_3) = \mathbb{P}(A_1)\mathbb{P}(A_2)\mathbb{P}(A_3)$.

Try to construct an example where three events are pairwise independent but not mutually independent.

Here is one potential starting point: Let A_1, A_2 be the respective results of flipping two fair coins. Can you come up with an event A_3 that works?