1 Student Life

In an attempt to avoid having to do laundry often, Marcus comes up with a system. Every night, he designates one of his shirts as his dirtiest shirt. In the morning, he randomly picks one of his shirts to wear. If he picked the dirtiest one, he puts it in a dirty pile at the end of the day (a shirt in the dirty pile is not used again until it is cleaned). When Marcus puts his last shirt into the dirty pile, he finally does his laundry, and again designates one of his shirts as his dirtiest shirt (laundry isn't perfect) before going to bed. This process then repeats.

(a) If Marcus has n shirts, what is the expected number of days that transpire between laundry events? Your answer should be a function of n involving no summations.

(b) Say he gets even lazier, and instead of organizing his shirts in his dresser every night, he throws his shirts randomly onto one of n different locations in his room (one shirt per location), designates one of his shirts as his dirtiest shirt, and one location as the dirtiest location. In the morning, if he happens to pick the dirtiest shirt, *and* the dirtiest shirt was in the dirtiest location, then he puts the shirt into the dirty pile at the end of the day and does not use that location anymore (it is too dirty now). What is the expected number of days that transpire between laundry events now? Again, your answer should be a function of n involving no summations.

2 Thanksgiving Feast

You are preparing a thanksgiving feast with n unique items of food to serve n customers, who each have a preference list of food items! To satisfy everyone, you made a list for each food item ordered by who you think that food item should be served to. All lists are independently and uniformly generated. To successfully satisfy a customer C with food item A, A must be at the top of C's preference list and C must be at the top of A's preference list. What is the expected number of successfully satisfied customers?

3 Tale Sum

You're listening to a local storyteller reciting a tall tale. The storyteller will vary the length of the tale based on how excitedly the crowd seems to be listening; you've determined that for all $t > 0$, the probability that the story goes on for exactly t minutes is $\frac{t}{(t+1)!}$. What is the expected length of the tale?

Hint: You can write $\frac{t}{(t+1)!}$ as $\frac{t+1}{(t+1)!} - \frac{1}{(t+1)!}$.

Hint 2: Recall the power series for e^x: $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$.