CS 70 Discrete Mathematics and Probability Theory
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DIS 6B

1 Expectation and Variance

This problem will give you some practice calculating expectations and variances of random variables. Suppose that the random variable X takes on 3 values, $10, 25, 70$. Suppose $\mathbb{P}[X = 10] = 0.5$, $\mathbb{P}[X = 25] = 0.2$, and $\mathbb{P}[X = 70] = 0.3$.

(a) What is $\mathbb{E}[X]$?

(b) What is $\mathbb{E}[X^2]$?

(c) What is $\text{var}(X)$?

2 Will I Get My Package?

A delivery guy in some company is out delivering n packages to n customers, where $n \in \mathbb{N}$, $n > 1$. Not only does he hand a random package to each customer, he opens the package before delivering it with probability $1/2$. Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbb{E}(X)$.

(b) Compute the variance $\text{var}(X)$.

3 Binomial Variance

Throw n balls into m bins uniformly at random. For a specific ball i, what is the variance of the number of roommates it has (i.e. the number of other balls that it shares its bin with)?

4 Rolling Dice

(a) If we roll a fair 6-sided die, what is the expected number of times we have to roll before we roll a 6? What is the variance?

(b) Suppose we have two independent, fair n-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance?