1 Variance

This problem will give you practice using the "standard method" to compute the variance of a sum of random variables that are not pairwise independent. Recall that $\text{var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

(a) A building has n floors numbered $1, 2, \ldots, n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each person gets off at one of the n floors uniformly at random (independently of everybody else). What is the *variance* of the number of floors the elevator *does not* stop at? (In fact, the variance of the number of floors the elevator *does* stop at must be the same, but the former is a little easier to compute.)

(b) A group of three friends has n books they would all like to read. Each friend (independently of the other two) picks a random permutation of the books and reads them in that order, one book per week (for n consecutive weeks). Let X be the number of weeks in which all three friends are reading the same book. Compute $\text{var}(X)$.

2 Correlation and Independence

(a) What does it mean for two random variables to be uncorrelated?

(b) What does it mean for two random variables to be independent?

(c) Are all uncorrelated variables independent? Are all independent variables uncorrelated? If your answer is yes, justify your answer; if your answer is no, give a counterexample.

3 Covariance

We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $\mathrm{cov}(X_1, X_2)$? Recall that $\mathrm{cov}(X_1, X_2) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$.