Lecture 1: Course Intro + Propositional Logic

Or: How I Learned to Stop Worrying and Love the Class

Welcome to CS 70!

What is this course?

- CS 70 is a math course.
  - Focus on proofs and justifications
  - Practice "mathematical thinking"
- Also an EECS course.
  - Probability, Cryptography, Graphs, ...
  - Applications throughout EECS

Why is this course?

- Learn to *think critically* and *argue clearly*
- See the building blocks of Computer Science

Adapting to this mindset can be hard — don't be discouraged, and do ask for help when you need it!

Staff Introductions

- Lecturers
- TAs
- Readers / Academic Interns

Logistics

Course website: `eecs70.org`

- Has homeworks, lecture notes, slides, etc.

Piazza: main avenue of communication

- Ask any questions here!
- Can make a private post or email `su19@eecs70.org` for more personal questions
- Will also post announcements here

Homework 0 (logistics) already out, Homework 1 coming soon

A Word on Homeworks

Intended to help you internalize the material.

Previous grading system focused on getting all the answers, even if you weren't learning anything.

This summer, new grading policy to address this:

- Graded on putting "reasonable effort" into each problem (see website for details)
- Translation: try all the problems, write down your thought process and where exactly you got stuck
- Readers still give detailed feedback
- Do still try your best on the homeworks — they're there for your benefit!

Lewis Carroll and Logic

(I) No one, who is going to a party, ever fails to brush his or her hair.
(II) No one looks fascinating, if he or she is untidy.
(III) Opium-eaters have no self-command.
(IV) Everyone who has brushed his or her hair looks fascinating.
(V) No one wears kid gloves, unless he or she is going to a party.
(VI) A person is always untidy if he or she has no self-command.

-Lewis Carroll, *Symbolic Logic*, 1896

Alice brushed her hair. What else do we know?

Propositions

A statement which is (unambiguously) true or false.
Basic building block of our logical system.

Examples:

- $2+2=4$ (True)
- I had pizza for breakfast this morning (False)
- Even integers are the sum of two primes[1] (???)

Not examples:

- A Pop-Tart is a sandwich (define sandwich)
- $x+2=7$ (what is x?)
- 17 (not making a claim)
- Jazz is better than Rock (personal preference)

[1]This is known as the Goldbach Conjecture

Propositional Formulae

Can combine propositions with logical operators:

- $P \wedge Q$ (Conjunction, "both P and Q")
- $P \vee Q$ (Disjunction, "at least one of P or Q")
- $\neg P$ (Negation, "not P")

String together for more complicated formulae:

- $P \wedge Q \wedge R$
- $(P \vee Q) \wedge (\neg P)$
- $(P \wedge Q \wedge R) \vee ((\neg P) \wedge (\neg Q) \wedge (\neg R))$

Use parentheses: $P \wedge Q \vee R$ is ambiguous!

Truth Tables

Would like to be able to compare formulae.
Ex: $P \vee Q$ vs $\neg((\neg P) \wedge (\neg Q))$

Idea: treat formulae as functions.

- Inputs are T/F values to each proposition
- Output is T/F value of overall formula
- Equivalent if have same *truth table*

For our example:

P	Q	$\neg P$	$\neg Q$	$\neg((\neg P) \wedge (\neg Q))$	$P \vee Q$
F	F	T	T	**F**	**F**
F	T	T	F	**T**	**T**
T	F	F	T	**T**	**T**
T	T	F	F	**T**	**T**

Our First Theorem

Claim: Can express any truth table as a formula.[2]

Example (on two variables):

P	Q	φ
F	F	T
F	T	F
T	F	T
T	T	F

φ not satisfied! Fix this.

$\varphi = ((\neg P) \wedge (\neg Q)) \vee (P \wedge (\neg Q))$

General Form:

- Disjunction (or) of many "and" clauses
- One clause for each satisfying assignment

[2]This means our language is *fully expressive*.

De Morgan's Laws

How does negation interact with $\wedge$ and $\vee$?

- $\neg(P \wedge Q)$
  "It is not the case that both P and Q are true"
- $(\neg P) \vee (\neg Q)$
  "Either P or Q must be false"

Saying the same thing! $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$

Try the same thing with $\vee$!

- $\neg(P \vee Q)$
  "It cannot be that either P or Q is true"
- $(\neg P) \wedge (\neg Q)$
  "Both P and Q are false"

Again the same! $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

An Interesting Corollary

Claim: Only needs $\vee$ and $\neg$ to be fully expressive.

Why? Follow this procedure:

1. Start with any truth table
2. Previous theorem gives formula φ
3. Use De Morgan's Laws to eliminate all $\wedge$s

End with equivalent formula using only $\neg$ and $\vee$!

Can instead eliminate all $\vee$s, leaving only $\wedge$ and $\neg$.

Implications

Even though we have enough to be fully expressive, having more operators is much more convenient.

One important symbol: implication ($\implies$)

P	Q	$P \implies Q$	$(\neg P) \vee Q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

Read: "P implies Q" or "if P then Q"
Equivalent to $(\neg P) \vee Q$

Wait a Second!

Why is "F $\implies$ T" true?
Think about "If it rains, the streets will be wet."
What if a fire hydrant breaks?

Alternative is biconditional $\iff$ ("if and only if")

P	Q	$P \iff Q$
F	F	T
F	T	F
T	F	F
T	T	T

$(P \iff Q) \equiv ((P \implies Q) \wedge (Q \implies P))$
Exercise: verify this with truth tables!

Converse and Contrapositive

How does $P \implies Q$ compare to $Q \implies P$?

P	Q	$P \implies Q$	$Q \implies P$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

Not the same! Think "if it rains, the streets will be wet" versus "if the streets are wet, it rained".

If want logical equivalence, need *contrapositive*:
$(\neg Q) \implies (\neg P)$

Think "if the streets are not wet, it did not rain".

A Well-Deserved Break

Listening to someone talk for an hour and a half is tough. Will include a ~4 minute break somewhere.

Good time to get to know your neighbors! Ask questions, form study groups, or just, ya know, be social :)

Today's discussion question:
Does pineapple belong on pizza?

Quantifiers

For any natural number x, define two propositions:

- $E(x)$ says that "x is even"
- $O(x)$ says that "x is odd"

How to say "0 is either even or odd"?
Easy enough: $E(0) \vee O(0)$

"Everything smaller than 3 is either even or odd"?
$(E(0) \vee O(0)) \wedge (E(1) \vee O(1)) \wedge (E(2) \vee O(2))$

"All natural numbers are either even or odd"?
O no.
Requires an infinitely-long formula :(
Need new tools.

Quantifiers

$\forall$ ("for all") and $\exists$ ("there exists")

Now write $(\forall n \in \mathbb{N})(E(n) \vee O(n))$

For a proposition $P(x)$[3] and a set S, we say

- $(\forall x \in S)P(x)$ if $P(x)$ holds for all $x \in S$
- $(\exists x \in S)P(x)$ if $P(x)$ holds for some $x \in S$

If S clear from context or doesn't matter[4], may omit

Multiple free variables needs multiple quantifiers!
Example: $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(x = y)$

Careful! $\forall$ and $\exists$ don't commute with each other.

[3] Here, x is a variable allowed to appear in P, called a "free variable"
[4] ~~or we're just lazy~~

Negation and Quantifers

How does negation interact with quantifiers?

- $\neg(\forall x\, P(x))$
  "$P(x)$ is not true for all x"
- $\exists x\, (\neg P(x))$
  "There is some x such that $P(x)$ doesn't hold"

Saying the same thing! $\neg(\forall x\, P(x)) \equiv \exists x\, (\neg P(x))$

What about with $\exists$?

- $\neg(\exists x\, P(x))$
  "There is no x satisfying $P(x)$"
- $\forall x\, (\neg P(x))$
  "$P(x)$ is false for all x"

Again the same thing! $\neg(\exists x\, P(x)) \equiv \forall x\, (\neg P(x))$

An Alternate Intuition

Recall De Morgan's Laws from earlier:

- $\neg(P \land Q) \equiv (\neg P) \lor (\neg Q)$
- $\neg(P \lor Q) \equiv (\neg P) \land (\neg Q)$

$\forall$ is like a conjunction ($\land$)
$(\forall x \in \mathbb{N}) P(x)$ is like $P(0) \land P(1) \land P(2) \land \ldots$

$\exists$ is like a disjunction ($\lor$)
$(\exists x \in \mathbb{N}) P(x)$ is like $P(0) \lor P(1) \lor P(2) \lor \ldots$

$\neg$ interacts with $\forall$ and $\exists$ like with $\land$ and $\lor$!

Lewis Carroll 2: Electric Boogaloo

Recall:

(I) No one, who is going to a party, ever fails to brush his or her hair. $P(x) \implies B(x)$
(II) No one looks fascinating, if he or she is untidy. $U(x) \implies (\neg F(x))$
(III) Opium-eaters have no self-command. $O(x) \implies (\neg S(x))$
(IV) Everyone who has brushed his or her hair looks fascinating. $B(x) \implies F(x)$
(V) No one wears kid gloves, unless he or she is going to a party. $K(x) \implies P(x)$
(VI) A person is always untidy if he or she has no self-command. $(\neg S(x)) \implies U(x)$

(Lewis Carroll 2: Electric Boogaloo) 2

(I) $P(x) \implies B(x)$	(II) $U(x) \implies (\neg F(x))$
(III) $O(x) \implies (\neg S(x))$	(IV) $B(x) \implies F(x)$
(V) $K(x) \implies P(x)$	(VI) $(\neg S(x)) \implies U(x)$

Alice brushed her hair. What else do we know?

$$B(x) \xRightarrow{IV} F(x) \xRightarrow{II(c)} \neg U(x) \xRightarrow{VI(c)} S(x) \xRightarrow{III(c)} \neg O(x)$$

So Alice looks fascinating, is not untidy, has self control, and is not an opium-eater.

Is she going to a party? Not enough information!

Fin

Next time: proofs!