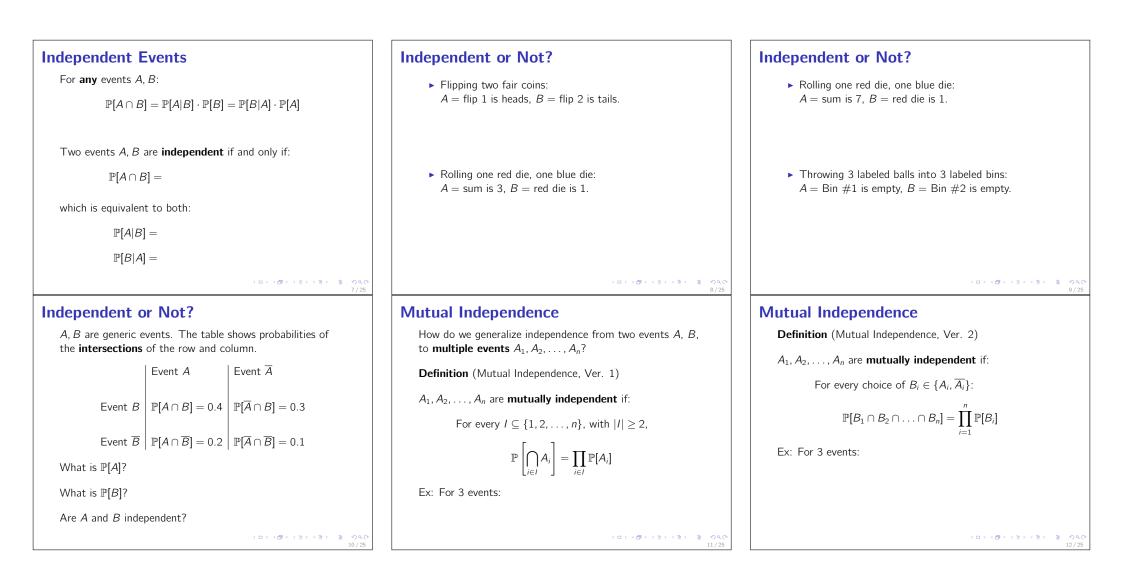
Intersections and Unions of Events CS 70, Summer 2019 Lecture 17, 7/23/19	 Last Time: Conditional Probability ▶ ℙ[A B]:restricting the sample space to B ℙ[A ∩ B] = ℙ[A B] ℙ[B] = ℙ[B A] ℙ[A] * "Total probability rule:" probability by disjoint cases * "Bayes' Rule": definition of conditional probability + total probability rule 	Computing Intersections For any events <i>A</i> , <i>B</i> : $\mathbb{P}[A \cap B] =$ What about any (three) events , <i>A</i> , <i>B</i> , <i>C</i> ? $\mathbb{P}[A \cap B \cap C] =$
Computing Intersections: Chaining For any events $A_1, A_2,, A_n$: $\mathbb{P}\left[\bigcap_{i=1}^n A_i\right] = \mathbb{P}[A_1] \cdot \mathbb{P}[A_2 A_1] \cdot \mathbb{P}[A_3 A_1 \cap A_2] \cdot$ Proof: Details are in the notes. General Idea: Key Insight: Treat $(A_1 \cap A_2 \cap \cap A_{n-1})$ as one event, treat A_n alone as another.	• Lets you "flip" the conditioning Drawing Cards I I draw 4 cards sequentially from a standard deck, without replacement. What is the probability that all 4 are clubs? $C_i =$ $\mathbb{P}[C_1 \cap C_2 \cap C_3 \cap C_4] =$	Drawing Cards II (Modified from notes.) I am dealt 5 cards. What is the probability that all five cards are the same suit, and none of them are face cards? $C_1 =$ For $2 \le i \le 5$: $C_i =$ $\mathbb{P}[C_1 \cap C_2 \cap C_3 \cap C_4 \cap C_5] =$
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A Weaker Idea: Pairwise Independence Definition (Pairwise Independence) $A_1, A_2,, A_n$ are pairwise independent if: For every $i \neq j$ in $\{1, 2,, n\}$: $\mathbb{P}[A_i \cap A_j] = \mathbb{P}[A_i] \cdot \mathbb{P}[A_j]$ Q: Does mutual imply pairwise? Q: Does pairwise imply mutual?	Using (Mutual) Independence: Coin Flips What is the probability that after n flips, we have k heads and $(n - k)$ tails?	Using (Mutual) Independence: Dice Rolls We roll <i>n</i> red dice and <i>n</i> blue dice. What is the probability that all the red dice are even, and all the blue dice are ≥ 5 ?
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Break	Unions of Events	Unions of Events
Back by popular demand	Same exact story as in counting	Same exact story as in counting
Would you rather only use spoons (no forks) or only use forks (no spoons) for the rest of your life?		
A joke Why was 6 afraid of 7? Because seven <i>ate</i> nine. Now, why was 7 afraid of 8?		
	$\mathbb{P}[A\cup B] =$	$\mathbb{P}[A \cup B \cup C] =$
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I roll a red die, a blue die, and a green die. What is the probability that at least one of these happen?

A) The red die's number is 3, or 4

B) The blue die's number is 5.

C) The green die's number is 1 or 6.

 $\mathbb{P}[A] =$

 $\mathbb{P}[B] =$

 $\mathbb{P}[C] =$

The Union Bound

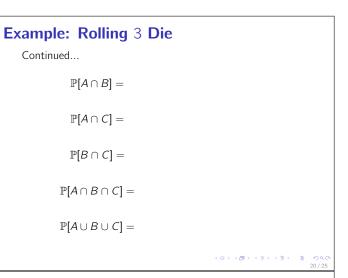
Q: What is the maximum possible value of the following?

 $\mathbb{P}[A_1 \cup A_2 \cup \ldots \cup A_n]$

A:

 $\mathbb{P}[A_1 \cup A_2 \cup \ldots \cup A_n]$ is always upper bounded by

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Union Bound Example: Rolling 3 Die

I roll a red die, a blue die, and a green die. What is **an easy upper bound** on the probability that at least one of these happen?

- A) The red die's number is 3 or 4.
- **B)** The blue die's number is 5.
- C) The green die's number is 1 or 6.

Principle of Inclusion and Exclusion

Same exact story as in counting...

For probability: Let $A_1, A_2, ..., A_n$ be events in our probability space. Denote $\{1, 2, ..., n\}$ by [n]. Then:

$$\mathbb{P}\left[\bigcup_{i=1}^{n} A_{i}\right] = \sum_{\{i\}\subseteq[n]} \mathbb{P}[A_{i}] - \sum_{\{i,j\}\subseteq[n]} \mathbb{P}[A_{i} \cap A_{j}] \\ + \sum_{\{i,j,k\}\subseteq[n]} \mathbb{P}[A_{i} \cap A_{j} \cap A_{k}] - \dots \\ \dots + (-1)^{n+1} \mathbb{P}[A_{1} \cap A_{2} \cap \dots \cap A_{n}]$$

Summary

- Computing event intersections = chaining conditional probabilities
 - Independent events = directly multiply probabilities
 - \blacktriangleright Mutual independence \neq pairwise independence
- Computing event unions = same exact strategy from counting!
 - ► Draw the Venn diagram for 2 events, 3 events
 - Principle of Inclusion-Exclusion for multiple events
- Union bound = worst case, the events are disjoint!



- ► Don't overthink it! Consider one thing at a time.
- Label your events!! Be cognizant of whether or not you are conditioning.
- If you have time, try a different strategy and see if it gets you the same answer (e.g. cases vs. complement)
- ► Try small examples to sanity-check your strategy!
- Practice, practice, practice!