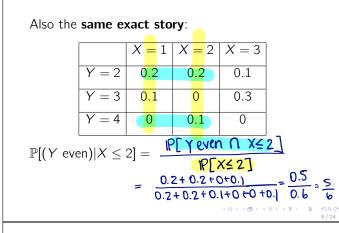
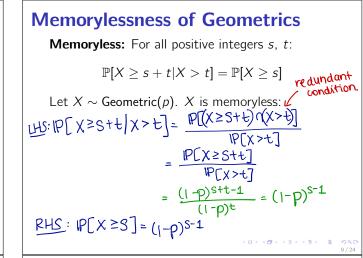


Conditional Distributions



Break

Which building on or near campus is your "spirit building"?



Expectation of a RV

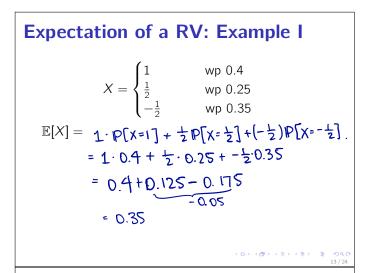
Also called the **mean** or **average** of a RV. Let X be a RV with values in A.

Its **expectation** is defined as:

$$\mathbb{E}[X] = \sum_{\alpha \in A} \alpha \cdot \mathbb{P}[X = \alpha]$$

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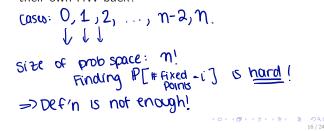
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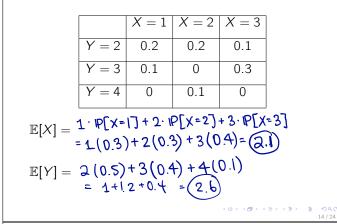
Mixing Up HW

n students turn in their HW, but I accidently mix them up. I return HW to the students, so that each mixup (**permutation**) is equally likely.

What is the expected number of students who get their own HW back?







Linearity of Expectation

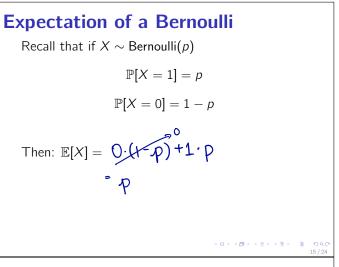
The definition of expectation isn't always easy to use. **Linearity** remedies this.

Theorem: Let $X_1, X_2, ..., X_n$ be RVs over the same probability space. They are **not necessarily independent.** Then:

$$\bigcirc \qquad \mathbb{E}[X_1 + \ldots + X_n] = \mathbb{E}[X_1] + \ldots + \mathbb{E}[X_n]$$

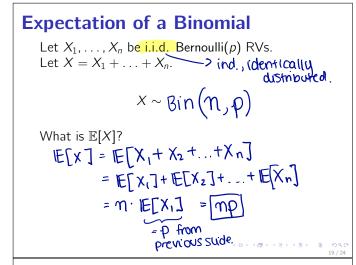
For constant
$$c$$
, $\mathbb{E}[cX_i] = c \cdot \mathbb{E}[X_i]$

Proof: Notes. Out of scope, but not a hard proof. Maybe formally go through it next lecture. Probably



Linearity: Example I

		X = 1	<i>X</i> = 2	<i>X</i> = 3		
	Y = 2	0.2	0.2	0.1		
	Y = 3	0.1	0	0.3		
	Y = 4	0	0.1	0		
From previous: $\mathbb{E}[X] = 2.1$, $\mathbb{E}[Y] = 2.6$. $\mathbb{E}[3X + 7Y] = \mathbb{E}[3X] + \mathbb{E}[7Y]$ $= 3 \mathbb{E}[X] + 7 \mathbb{E}[Y]$ $= 3 \cdot 2.1 + 7 \cdot 2.6$ = 6.3 + 18.2 = 24.5						



A Note on Symmetry

 C_i = indicator for the *i*-th card being an ace.

$$\mathbb{P}[C_i=1] = \frac{1}{13}$$

Now, imagine I draw the entire deck.

$$\mathbb{E}[C_1 + C_2 + \ldots + C_{52}] = 4$$
of aces I get:
= 4, always
Using this, for any *i*, what is $\mathbb{E}[C_i]$?
 $\frac{4}{52} = \frac{1}{13}$

Linearity: Example II
I draw two cards from a standard deck.
What is the expected number of aces I get?
Attempt #1: Use the definition.

$$E[A] = 0 \cdot P[A=0]^{+} | \cdot P[A=1] + 2 \cdot P[A=2]$$

$$P[A=1] = P[no_{ace}^{+}, ace] + P[ace, ace]$$

$$= (\frac{48}{52})(\frac{4}{51}) + (\frac{4}{52})(\frac{48}{51}) = \frac{384}{52 \cdot 51}$$

$$P[A=2] = (\frac{4}{52})(\frac{3}{51}) = \frac{12}{52 \cdot 51} = \frac{408}{52 \cdot 51} = \frac{8}{52}$$

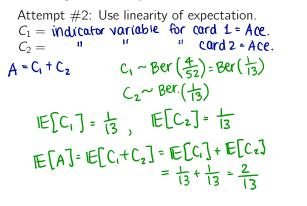
$$= \frac{2}{13}$$

Linearity: Mixing Up HW

(From notes.)
Same HW setup as before with *n* students.
Want, Expected # of fixed points.

$$S_i = \text{indicator variable for students i getting
their own HW.
 $S_i \sim Ber(\frac{1}{n})$
 $S = # of fixed points (students wins get
 $S = S_i + S_2 + ... + S_n$
 $E[S] = E[S_i + S_2 + ... + S_n]$
 $= H: E[S_i] = N(\frac{1}{n}) \in 1$$$$

Linearity: Example II



Summary

 Joint distribution: multiple RVs. Can still be defined for non-independent RVs.

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- Ideas of independence, conditional probability same as before.
- **Expectation** describes the **weighted average** of a RV.
- For more complicated RVs, break down into smaller parts (e.g. indicator variables) and use linearity