Variance

CS 70, Summer 2019

Lecture 21, 7/30/19

Two Games

Game 1: Flip a coin 10 times. For each Head, you win 100. For each Tail, you lose 100.

Expected Winnings on Flip i:

Expected Winnings After 10 Flips:

Two Games

Game 2: Flip a coin 10 times. For each Head, you win 10000. For each Tail, you lose 10000.

Expected Winnings on Flip i:

Expected Winnings After 10 Flips:

Q: Which game would you rather play?

Definition of Variance

The key difference is the **variance**.

Variance is the **expected "distance" to mean.**

Let X be a RV with $\mathbb{E}[X] = \mu$. Then:

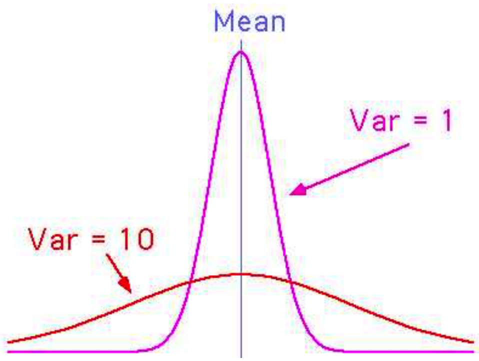
$\text{Var}(X) =$

Alternate Definition

We can use **linearity of expectation** to get an alternate form that is often **easier to apply**.

$\text{Var}(X) =$

Variance: A Visual



Variance of a Bernoulli

Let $X \sim \text{Bernoulli}(p)$.
Then $\mathbb{E}[X] =$

What is X^2? $\mathbb{E}[X^2]$?

$\text{Var}[X] =$

Variance of a Dice Roll

What is the variance of a single 6-sided dice roll?

$R =$

What is R^2?

Variance of a Dice Roll

$\mathbb{E}[R^2] =$

$\text{Var}(R) =$

Variance of a Geometric

Know the variance; proof optional, but good practice with **manipulating RVs**.

Let $X \sim \text{Geometric}(p)$.
Strategy: Nice expression for $p \cdot \mathbb{E}[X^2]$

Variance of a Geometric II

From the distribution of X, we know:

From $\mathbb{E}[X]$, we know:

Variance of a Geometric III

Recall $\mathbb{E}[X] = \frac{1}{p}$.

Variance of a Poisson

Same: **know the variance;** proof optional, but good practice with **functions of RVs**.

Let $X \sim \text{Poisson}(\lambda)$.
Strategy: Compute $\mathbb{E}[X(X-1)]$.

Variance of a Poisson II

Use $\mathbb{E}[X(X-1)]$ to compute $\text{Var}(X)$.

Break

Would you rather only wear sweatpants for the rest of your life, or never get to wear sweatpants ever again?

Properties of Variance I: Scale

Let X be a RV, and let $c \in \mathbb{R}$ be a constant.
Let $\mathbb{E}[X] = \mu$.

$\text{Var}(cX) =$

Properties of Variance II: Shift

Let X be a RV, and let $c \in \mathbb{R}$ be a constant.
Let $E[X] = \mu$.
Then, let $\mu' = \mathbb{E}[X + c] =$

$\text{Var}(X + c) =$

Example: Shift It!

Consider the following RV:

$$X = \begin{cases} 1 & \text{w.p. } 0.4 \\ 3 & \text{w.p. } 0.2 \\ 5 & \text{w.p. } 0.4 \end{cases}$$

What is $\text{Var}(X)$? Shift it!

Sum of Independent RVs

Let $X_1, \ldots, X_n$ be independent RVs. Then:

$$\mathrm{Var}(X_1 + \ldots + X_n) = \mathrm{Var}(X_1) + \ldots + \mathrm{Var}(X_n)$$

Proof: Tomorrow!

Today: Focus on applications.

Variance of a Binomial

Let $X \sim \mathrm{Bin}(n, p)$. Then,

$$X =$$

Here, $X_i \sim$

$\mathrm{Var}(X) =$

Sum of Dependent RVs

Main strategy: **linearity of expectation** and **indicator variables**

Useful Fact:

$$(X_1 + X_2 + \ldots + X_n)^2 =$$

HW Mixups (Fixed Points)

(In notes.) n students hand in HW. I mix up their HW randomly and return it, so that every possible mixup is **equally likely**.

Let $S = \#$ of students who get their own HW.

Last time: defined $S_i =$

$$S_i \sim$$

Using **linearity of expectation**:
$\mathbb{E}[S] =$

HW Mixups II

Using our **useful fact**:
$\mathbb{E}[S^2] =$

HW Mixups III

What is S_i^2? $\mathbb{E}[S_i^2]$?

For $i \neq j$, what is $S_i S_j$? $\mathbb{E}[S_i S_j]$?

HW Mixups IV

Put it all together to compute $\mathrm{Var}(X)$.

Summary

Today:

- Variance measures how far you **deviate from mean**
- Variance is additive for **independent RVs**; proof to come tomorrow
- Use **linearity of expectation** and **indicator variables**