The Poisson Arrival Process

CS 70, Summer 2019

Bonus Lecture, 8/14/19

Poisson Distribution: Review

Values:

Parameter(s):

$\mathbb{P}[X = i] =$

$\mathbb{E}[X] =$

$\text{Var}[X] =$

Poisson Over Time

Let $B_1 \sim \text{Poisson}(\lambda)$ be the number of bikes that are stolen on campus in one hour. (Go bears?)

What is the distribution of $B_{2.5}$, the number of bikes that are stolen on campus in two hours?

Rate over time $T =$

Adding Poissons: Review

Let $T_1 \sim \text{Poisson}(\lambda_1)$ be the number of particles detected by Machine 1 over 3 hours.

Let $T_2 \sim \text{Poisson}(\lambda_2)$ be the number of particles detected by Machine 2 over 4 hours.

The machines run **independently**.

What is the distribution of $T_1 + T_2$?

Adding Poissons: Twist?

What is the distribution of the **total number of particles detected across both machines** over 5 hours?

Decomposing Poissons

Let $T \sim \text{Poisson}(\lambda)$ be the number of particles detected by a machine over one hour.

Each particle behaves **independently** of others.

Each detected particle is an α-particle with probability p, and a β-particle otherwise.

Let T_α be the number of α-particles detected by a machine over one hour. What is its distribution?

Decomposing Poissons

Let T_α be the number of α-particles detected by a machine over one hour. What is its distribution?

How about T_β, the number of β-particles?

Independence?

Are T_α and T_β independent?

Decomposing Poissons Remix

Now there are 3 kinds of particles: α, β, γ.

Each detected particle behaves independently of others, and is α with probability p, β with probability q, and γ otherwise.

$$T_\alpha \sim$$

$$T_\beta \sim$$

$$T_\gamma \sim$$

Punt: T_α, T_β, T_γ are **mutually independent**.
Sanity Check: $T_\alpha + T_\beta + T_\gamma \sim$

Exponential Distribution: Review

Values:

Parameter(s):

$\mathbb{P}[X = i] =$

$\mathbb{E}[X] =$

$\text{Var}[X] =$

Break

If you could rename the Poisson RV (or any RV for that matter), what would you call it?

Poisson Arrival Process Properties

We'll now work with a specific setup:

- There are **independent** "arrivals" over time.
- The time between consecutive arrivals is $\text{Expo}(\lambda)$. We call λ the **rate**.
  Times between arrivals also **independent**.
- For a time period of length t, the **number of arrivals** in that period is $\text{Poisson}(\lambda t)$.
- Disjoint time intervals have independent numbers of arrivals.

Poisson Arrival Process: A Visual

Transmitters I

A transmitter sends messages according to a Poisson Process with hourly rate λ.

Given that I've seen 0 messages at time t, what is the expected time until I see the first?

Transmitters I

How many messages should I expect to see from 12:00-2:00 and 5:00-5:30?

Transmitters II: Superposition

Transmitters A, B sends messages according to Poisson Processes of rates λ_A, λ_B respectively. The two transmitters are **independent**.

We receive messages from both A and B.

What is the expected amount of time until the first message from either transmitter?

Transmitters II: Superposition

Transmitters A, B sends messages according to Poisson Processes of rates λ_A, λ_B respectively. The two transmitters are **independent**.

We receive messages from both A and B.

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Transmitters II: Superposition

If the messages from A all have 3 words, and the messages from B all have 2 words, how many words do we expect to see from 12:00-2:00?

Kidney Donation: Decomposition

My probability instructor's favorite example...

Kidney donations at a hospital follow a Poisson Process of rate λ per day. Each kidney either comes from blood type A or blood type B, with probabilities p and $(1-p)$ respectively.

Kidney Donation: Decomposition

If I have blood type B, how long do I need to wait before receiving a compatible kidney?

Say I just received a type A kidney.
The patient receiving a type A kidney after me is expected to live 50 more days without a kidney donation. What is the probability they survive?

Kidney Donation: Decomposition

Now imagine kidneys are types A, B, O with probabilities p, q, $(1-p-q)$, respectively.

If I have type B blood, I can receive both B and O. How many compatible kidneys do I expect to see over the next 3 days?

Summary

When working with **time**, use Expo(λ) RVs.

When working with **counts**, use Poisson(λ) RVs.

Superposition: combine independent Poisson Processes, **add** their rates.

Decomposition: break Poisson Process with rate λ down into rates $p_1\lambda$, $p_2\lambda$, and so on, where p_i's are probabilities.