The Poisson Arrival Process

CS 70, Summer 2019

Bonus Lecture, 8/14/19

Adding Poissons: Review

Let $T_1 \sim \text{Poisson}(\lambda_1)$ be the number of particles detected by Machine 1 over 3 hours.

Let $T_2 \sim \text{Poisson}(\lambda_2)$ be the number of particles detected by Machine 2 over 4 hours.

The machines run **independently**.

What is the distribution of $T_1 + T_2$?

Poisson Distribution: Review

Values:

Parameter(s):

 $\mathbb{P}[X = i] =$

 $\mathbb{E}[X] =$

Var[X] =

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Adding Poissons: Twist?

What is the distribution of the **total number of particles detected across both machines** over 5 hours?

Poisson Over Time

Let $B_1 \sim \text{Poisson}(\lambda)$ be the number of bikes that are stolen on campus in one hour. (Go bears?)

What is the distribution of $B_{2.5}$, the number of bikes that are stolen on campus in two hours?

Rate over time T =

Decomposing Poissons

Let $T \sim \text{Poisson}(\lambda)$ be the number of particles detected by a machine over one hour.

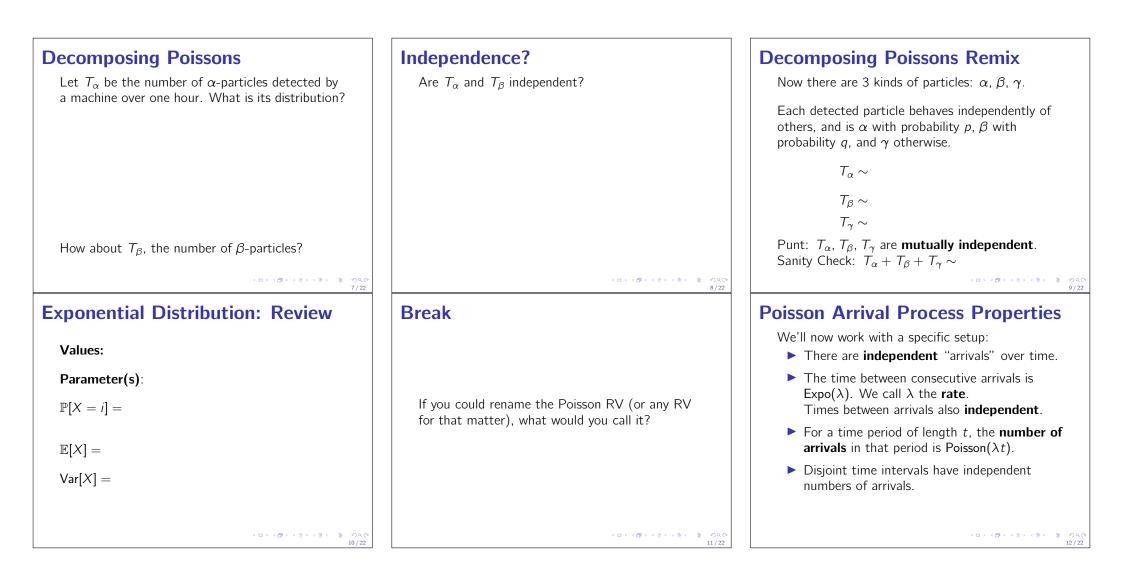
Each particle behaves **independently** of others.

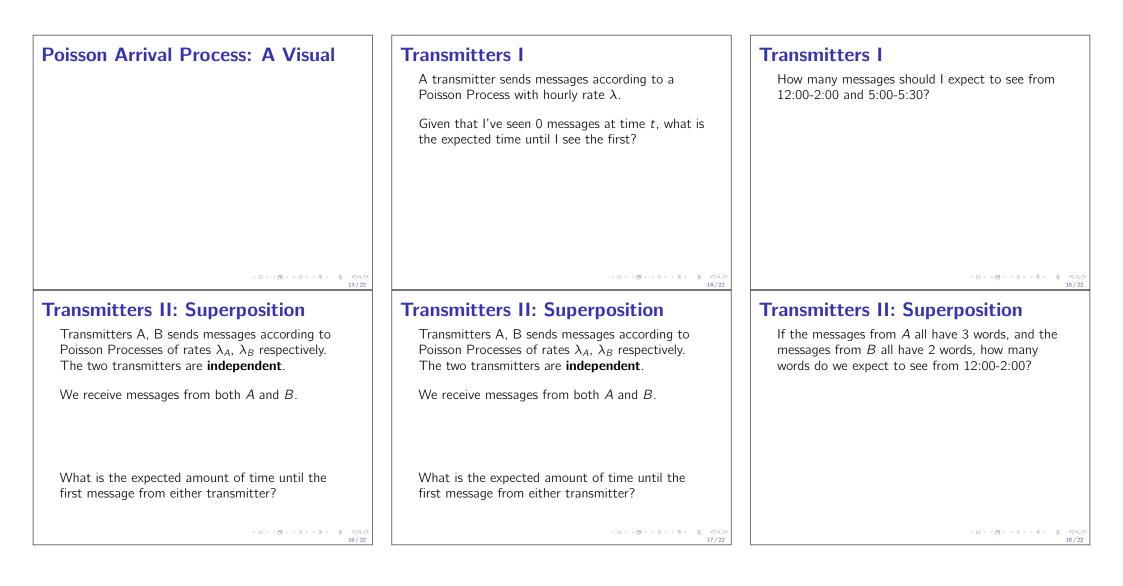
Each detected particle is an α -particle with probability *p*, and a β -particle otherwise.

Let T_{α} be the number of α -particles detected by a machine over one hour. What is its distribution?

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Kidney Donation: Decomposition	Kidney Donation: Decomposition	Kidney Donation: Decomposition
My probability instructor's favorite example Kidney donations at a hospital follow a Poisson Process of rate λ per day. Each kidney either comes from blood type A or blood type B , with probabilities p and $(1 - p)$ respectively.	If I have blood type B, how long do I need to wait before receiving a compatible kidney? Say I just received a type A kidney. The patient receiving a type A kidney after me is expected to live 50 more days without a kidney donation. What is the probability they survive?	Now imagine kidneys are types A, B, O with probabilities p , q , $(1 - p - q)$, respectively. If I have type B blood, I can receive both B and O. How many compatible kidneys do I expect to see over the next 3 days?
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Summary

When working with **time**, use $Expo(\lambda)$ RVs.

When working with **counts**, use $Poisson(\lambda)$ RVs.

Superposition: combine independent Poisson Processes, **add** their rates.

Decomposition: break Poisson Process with rate λ down into rates $p_1\lambda$, $p_2\lambda$, and so on, where p_i 's are probabilities.

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